Time series investigations on an experimental system driven by phase transitions

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(Received 6 February 1997)

In the present paper the analysis of an intermittent time series obtained from a nonlinear system is presented. The setup is driven by phase transitions and has no external periodic driving. The observed states range from stable ones over periodicity and intermittence to chaotic behavior. The data in the intermittent regime are analyzed with respect to the optical shape, the frequency distribution of the laminar lengths, and the γ parameter. The results of these analyses are compatible with the interpretation of the presence of intermittence type II in the system under investigation. Additionaly the reconstruction of the radius dynamics of the experimental data is attempted. The corresponding results also indicate type II intermittence. Some weight is put on figuring out the principal problems, arising in the reconstruction of the Neĭmark-Sacker dynamics from a scalar time series. [S1063-651X(97)04707-7]

PACS number(s): 64.60.Ak, 47.20.Ky,44.90.+c

I. INTRODUCTION

There are several ways by which a system can change its dynamical behavior. A special class of these paths are transitions from periodicity to chaos. And within this class there are three, which have been investigated in detail by Pommeau and Manneville [1]. These three transition types are called intermittency I, II, and III. Type I and type III intermittency are observed rather frequently in a large variety of nonlinear systems [2–6]. For type II intermittence, however, only a few papers can be found on experimental results [7,8], but, nevertheless, a variety of theoretical papers exist [9,10,1]. In the cases dealing with experimental systems, most of them are driven by an external, periodic force, except the one reported by Herzel, Plath, and Svensson [7]. The present paper reports on observations on a nonlinear dynamical system that is also not driven by an external force and which yields strong evidence for type II intermittence.

In Sec. II the experimental setup is described. In Sec. III the experimental results are shown. Sections IV and V deal with the different analyzing methods applied to the data. In Sec. VI of the paper the results are summarized and a short discussion is given.

II. THE EXPERIMENT

The system comprises a heating region, in which a medium (at present water) is vaporized, and a cooling region, in which condensation of the medium takes place.

The quantity measured primarily is the pressure in the air buffer (see Fig. 1), which in turn is a measure for the position x of the phase boundary between the vapor phase and the liquid phase of the medium. As long as the pressure change is kept small enough, the pressure is a good approximation for the position x. In the general case this relation is nonlinear, however this results only in a distortion of the measured pressure signal but not in the dynamics of the system.

In this setup there are a lot of relevant parameters, such as the distance between the heating and the cooling region, the material of the heated tube (the main property in this case seems to be the thermal conductivity of the heated tube), the diameter of the tube, its wall thickness, the liquid that is vaporized, and, to some extent, the temperature of the cooling bath. There is also some influence from the length of the water column and the pressure in the air buffer.

Parameters which were varied systematically in the experiments are the spacing between the heating and the cooling region, the cooling temperature, the amount of water in the system, and the heating power. In this paper, however, only the effects of changing the heating power are discussed, while the other parameters were kept as constant as possible during the experiments.

III. EXPERIMENTAL RESULTS

The principal scenario, described below, can be observed with all parameters fixed except the heating power. However, for some parameter sets not all of the described states could be identified. In the case of low heating power, only heat conduction from the hot to the cold region in the liquid medium takes place. In this range of heating powers the heating region is not hot enough to enable vaporization of the medium. When the power is raised above a certain threshold, vaporization begins. The volume of the gaseous phase is much larger than the one of the liquid, thus pushing the position x of the meniscus between gas and liquid towards the cooling region. At moderate heating power the position x settles at a static value. Nevertheless, even with this static position of the meniscus, the equilibrium between the gaseous and the liquid phase is a dynamic one. The raising of the power over a second threshold leads to (bi)periodic os-

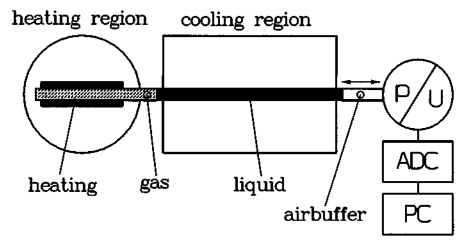

FIG. 1. Not to scale scheme of the experimental setup.

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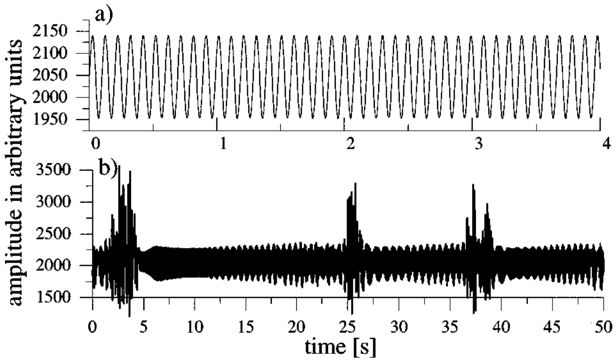

FIG. 2. (a) Periodic and (b) intermittent signal cuts.

cillations of the meniscus. Again there is a certain range of the power values for which these oscillatory modes are stable. In the case of even higher powers the oscillations become unstable and are interrupted by bursts. The system is in an intermittent state. With further increasing power the bursts become more and more frequent and finally the system reaches a chaotic state.

The parameter varied primarily is the heating power of the heater, however it seems to be more reasonable to identify the temperature of the heater as the control parameter of the system.

For the parameter set corresponding to the data reported in the following sections of the paper the transition from a stable meniscus position to an oscillatory state occurred at a temperature $T=535$ K. Intermittency was observed above $T=773$ K and the chaotic regime was reached at $T=923$ K. Representative signals for the periodic and the intermittent temperature range are shown in Fig. 2.

IV. ANALYSIS OF THE TRANSITION BETWEEN THE STATIC AND THE PERIODIC STATE

When the temperature rises above the threshold temperature (in the present case $T_S=535$ K) an oscillation of the position x starts. Within the experimental errors the frequency of the oscillation is independent of the difference $\Delta T=T-T_S$. However with increasing ΔT the amplitude of the oscillation increases. Figure 3 shows the amplitude as function of ΔT. The dashed line is a fit of a square root function to the data. Such a functional relation between the amplitude of the oscillation and the control parameter ΔT is typical for a supercritical Hopf bifurcation.

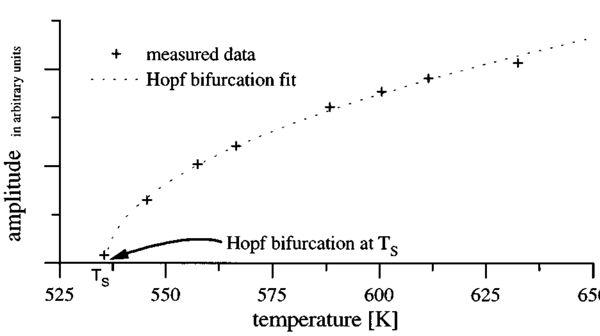

FIG. 3. Amplitudes of the harmonic oscillation.

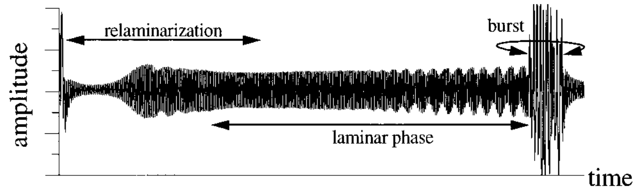

FIG. 4. Characteristic phases of an intermittent signal.

V. EXAMINATION OF THE INTERMITTENT REGIME

A. Visual inspection of the time series

A typical cut of the intermittent time series is shown in Fig. 4. It exhibits the three major components of each intermittent time series. There is a laminar region, which is interrupted by an irregular burst and after this burst a new laminar phase occurs, which is preceded by a relaminariztion period during which the signal recovers from the irregular burst towards the periodic oscillations. From the form of the signal during the laminar phase type III intermittency can be clearly ruled out for the present case, since for this type a second subharmonic oscillation with increasing amplitude occurs [9], which cannot be found in the observed time series. So from the standard intermittence models only types I and II remain.

Although this is not rigorous proof, the time series looks very similar to those reported from systems exhibiting type II intermittence i.e., in the theoretical paper of Richetti, Argoul, and Arneodo [10] or the experimental one of Herzel, Plath, and Svensson [7]. Some more information can be obtained from the analysis in the following subsection.

B. Distribution of the laminar length

In the standard cases of intermittent systems the scaling behavior of the length of the frequency $P(N)$ of the laminar length N can be derived from the nature of the underlying dynamical processes, i.e., in which way the oscillation loses its stability.

Since type III can be excluded, as mentioned in the preceding paragraph, only type I and II will be regarded now. For type I intermittence the presence of a sharp cutoff length N_{max} is necessary. N_{max} is the maximum laminar length that can be observed. In the normally given examples the $P(N)$ distribution is U shaped in the region $0<N<N_{max}$. Figure 5 shows the distribution of the laminar length for the investigated system measured at a heating power of $P=120$ W.

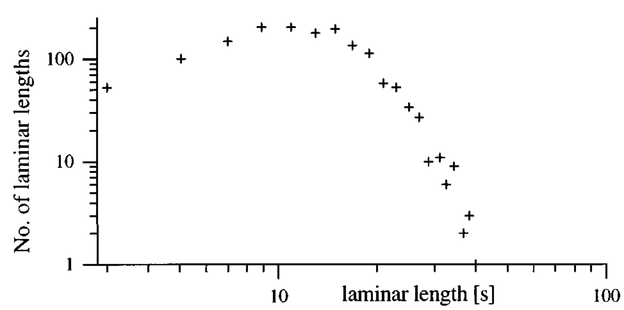

FIG. 5. Log-log plot of the distribution of laminar lengths (at $P=120$ W).

This plot does not give any evidence for the presence of a cutoff length.

In the case of type II intermittence normally $P(N)\sim N^{-2}$ is expected [9] which was also observed in the present experiment [11]. However, in most cases, this scaling law cannot be seen in the spectra as shown in Fig. 5. But this special power law is the result of the assumption of a uniform reinjection probability after the burst. This argument holds too for type I intermittence; but the reinjection probability only affects the shape of the $P(N)$ distribution and not such features as the presence of a cutoff length. Thus the only remaining possibility of the three standard types is type II intermittence, which will be looked at in some detail now.

Assuming a uniform reinjection probability is the most simple assumption that can be made for the reinjection in phase space. However this assumption does not reveal the structure of the observed data. The next more complicated reinjection distribution $P(\theta,r)$ is to assume still isotropy for the angular coordinate [for a phase space with cylindrical coordinate (θ,r) system], but a normal distribution around c in the r direction

$$P(r)\sim r\exp\left(-\frac{(c-r)^2}{2\,\sigma^2}\right).$$

The iteration map corresponding to type II intermittence is given by

$$r_{n+1}=(1+\mu)r_n+r_n^3.$$

This map has to be iterated starting with the value r_1 given by the reinjection mechanism. The laminar phase lasts N iterations until r_{N+1} exceeds a given value R. After integration the following relation between the starting value r and N can be found:

$$r(N)=\left(\frac{\mu}{f_N}\right)^{1/2}, \quad \text{with } f_N=(\mu+R^2)e^{2\mu N}-R^2$$

$$\Rightarrow\left|\frac{dr}{dN}\right|=\frac{\mu^{3/2}(f_N+R^2)}{f_N^{\,3/2}}.$$

By simple phase space considerations it can be found that the probability $P(N)$, to find a certain number of iterations, is related to the reinjection probability $P(r)$ by

$$P(N)=P(r)\left|\frac{dr}{dN}\right|.$$

Combining the last two equations yields

$$P(N)\sim(f_N+R^2)\left(\frac{\mu}{f_N}\right)^2 e^{-(\sqrt{\mu/f_N}-c)^2/2\,\sigma^2} \tag{1}$$

This is the expression for the probability to find a laminar length N when the reinjection process is isotropic in the θ coordinate but normally distributed in the r direction around a fixed distance c. Figure 6 shows the fit of Eq. (1) to the observed data (+=experimental data, solid line=fit of type II intermittency model). It can be seen that the fit describes the observed data in a satisfactory manner.

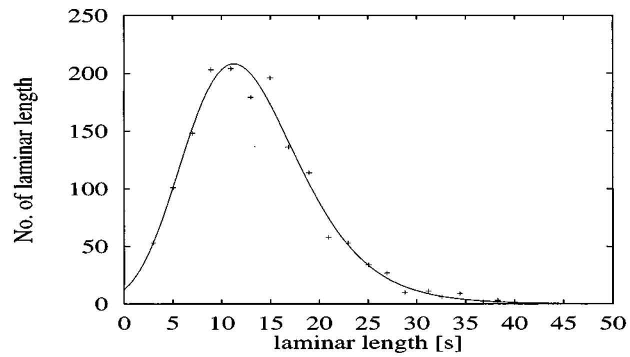

FIG. 6. Distribution of laminar lengths frequency.

Therefore the data are compatible with the intermittency type II dynamics with the assumption of a nonuniform reinjection probability. However, this is not a handicap because the assumption of a uniform reinjection is too simple for real processes. Also in the experiment of Herzel, Plath, and Svensson [7] and in the theoretical investigation of Richetti, Argoul, Arneodo [10], nonuniform reinjection probabilities are necessary to describe the distribution of the laminar length.

It should be mentioned at this point that the above assumption about the reinjection probability is not the only one that is compatible with the data. For example, if the distribution used by Herzel, Plath, and Svensson [7] is applied to the data, a good agreement is found, too. However this case is, to some extent, still one step further away from the assumption of uniformity since the isotropy in the θ coordinate is given up and a normally distributed reinjection around one point in phase space is assumed.

C. Mean laminar length and γ parameter

For intermittence I and II theoretical calculations give an expression for the mean laminar length $\overline{\tau}$ that is independent of the special functional dependence of the reinjection process. The corresponding relation for type II is $\overline{\tau}\sim\mu^{-1}$. However, it is rather problematic to determine $\overline{\tau}$ from the observed time series, since by depending on the reinjection mechanism there exists a rather large probability for short laminar length. The problem of determining the length of the laminar length consists of at least two contributions. First it is necessary to observe a minimum number of oscillations in order to see a laminar period. The second aspect is due to the problem to determine exactly when the relaminarization is finished and the laminar period starts. Therefore the errors in these rather frequent short laminar lengths are quite large.

A more robust quantity is the so called γ parameter [12] that is related to $\overline{\tau}$. The γ parameter measures the fraction of the turbulent phases compared to the length of the whole time series

with $\overline{\beta}=$(mean length of bursts)

and

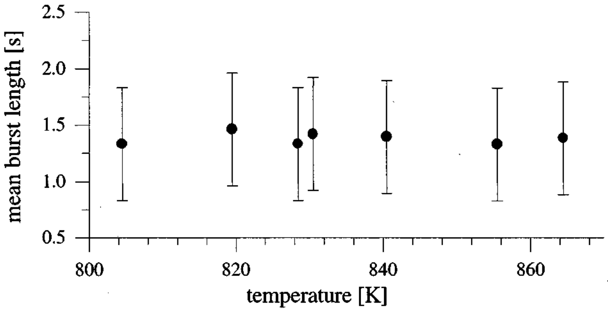

FIG. 7. Temperature dependence of the mean burst lengths $\overline{\beta}$.

$$\overline{\beta}+\overline{\tau}=\frac{t_{\text{total}}}{(\text{number of bursts})}$$

$$\Rightarrow\gamma=\frac{t_{\text{turbulent}}}{t_{\text{total}}}=\frac{\overline{\beta}}{\overline{\beta}+\overline{\tau}}.$$

From the experiment it can be found that $\overline{\beta}$ is nearly constant for moderate changes in the heating temperature (see Fig. 7). Assuming further $\overline{\tau}=a\mu^{-r}$ gives

$$\gamma(\mu)=\frac{\overline{\beta}}{\overline{\beta}+\overline{\tau}}=\left[1+\left(\frac{\overline{\tau}}{\overline{\beta}}\right)\right]^{-1}$$
$$=\left(1+\frac{a}{c\mu^{r}}\right)^{-1}=\left(1+\frac{d}{\mu^{r}}\right)^{-1},$$

with

$$d=\frac{a}{c},\quad \overline{\beta}=\text{const.} \tag{2}$$

So by fitting Eq. (2) to the experimentally determined γ-parameters results in a value for r. In the experiment the relative difference $(T-T_{cr})/T_{cr}$ is interpreted as the control parameter μ; i.e., since the measured quantities are γ and T the critical temperature T_{cr} is also obtained from the fit. A corresponding plot is shown in Fig. 8. Within the error bars

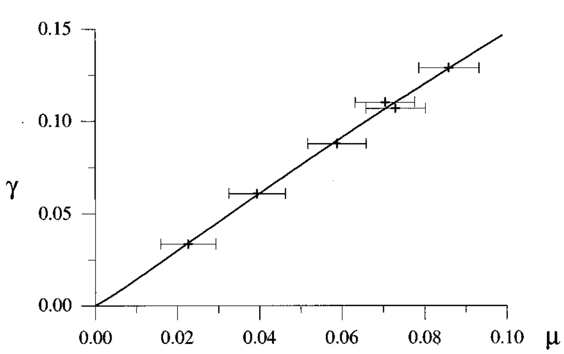

FIG. 8. Fit of Eq. 2 to the experimental γ parameters ($r=1.07$, $d=0.49$, $T_{cr}=773.75$ K). The error bars correspond to temperature variation during the laminar phases.

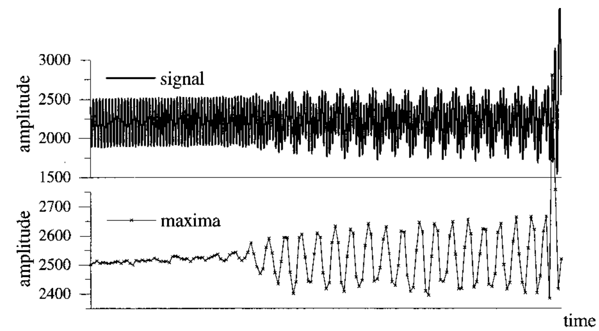

FIG. 9. Time series (upper plot) and envelope sampled by maxima (lower plot).

$r=1$ and thus the relation $\overline{\tau}\sim\mu^{-1}$ is obtained for the observed data.

D. Examination of the r dynamics via first-return maps

In cases where systems are driven by an external periodical signal, a canonical choice for the sampling frequency for a Poincaré plot is given by the frequency of the drive. In the present case however there is no external periodic drive (the system is driven by the energy flow between the heater and the cooling reservoir). The only remaining ''clock generator'' is given by the frequency of the faster oscillation during the laminar phases. The lower part of Fig. 9 shows such a plot of the envelope of the laminar phase, sampled at the instances of the maxima of the faster oscillation. Plotting the maximum x_{n+1} against maximum x_n results in the so called first-return map (Fig. 10). This plot shows a spiral-like behavior as is typical for type II intermittence. Extracting from such a plot the corresponding r_n values and plotting r_{n+1} against r_n should reveal the radius dynamics of the system under investigation. In practice, however, this attempt is somewhat problematic for reasons that are described in some detail now.

For this purpose the map Neĭmark-Sacker bifurcation, which is essential for type II intermittency, is iterated

$$r_{n+1}=(1+\mu)r_n+ar_n^3, \tag{3}$$

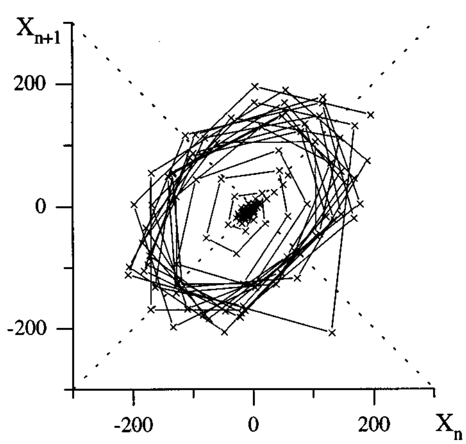

FIG. 10. First-return map of the laminar signal.

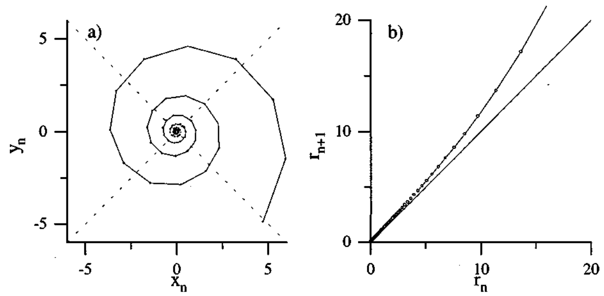

FIG. 11. (a) Poincaré plot and (b) radius dynamics of the iterated Neĭmark-Sacker-map [Eq. (3)].

$$\phi_{n+1}=\phi_n+\theta+O(r_n^2). \tag{4}$$

One obtains Cartesian coordinates by

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix}=\begin{pmatrix} r_n\sin(\phi_0+n\theta) \\ r_n\cos(\phi_0+n\theta) \end{pmatrix} \tag{5}$$

Figure 11 shows the corresponding plots for y_n vs x_n and r_{n+1} vs r_n. Now it is assumed that in an experiment the observed data are given by the x_n values from Eq. (5). A reconstruction of the Poincaré plot is made by plotting x_n vs x_{n+1} [see Fig. 12(a)]. It is clearly seen that the plot resembles the original spiral, but it exhibits strong distortions. Extracting r_n values from this spiral results in Fig. 12(b). The reason for this distortions is the transition from the point series given by the coordinate pairs from Eq. (5) to the one given by the pairs

$$\begin{pmatrix} x_n \\ x_{n+1} \end{pmatrix}=\begin{pmatrix} r_n\sin(\phi_0+n\theta) \\ r_{n+1}\sin[\phi_0+(n+1)\theta] \end{pmatrix}. \tag{6}$$

Inspecting the reconstructed spiral plot, it is recognized that only one point is located in the upper left and the lower right quadrant per spiral turn. This is due to the fact that these quadrants are populated by those points where the x_n data change sign. It is also recognized that the spiral is elongated along the diagonal. A possible correction is therefore given by rotating and stretching the original reconstruction. By such a linear mapping the plots can be improved; something like the plots shown in Figs. 12(c) and 12(d) seems to be the optimum. The remaining ''bumpy'' structure in the r_n plots expresses the fact that a nonlinear error is introduced by replacing $\cos(\theta_n)$ by $\sin(\theta_{n+1})$. Such behavior is also seen in the experimental data of Sreenivasan and Ramshankar [12].

Some further improvement can be achieved, evaluating only the data corresponding to a fixed direction in the (r,θ) plane. Applying such a correction to the experimental r_n data of Fig. 12, Fig. 13(a) is obtained. The points still exhibit some scattering. One reason for this is that in accordance with the exact Neĭmark-Sacker bifurcation dynamics, different from Eq. (4), $\theta=\phi_{n+1}-\phi_n$ is not fixed but slightly varying with n. For that reason, in the evaluation, not only points for a fixed direction can be used but a small range around this direction has to be accepted for the evaluation. For that reason, in Fig. 13(b) a smoothing for neighboring points is applied. In Fig. 13(c) the r_n dynamics of Eq. (3) are fitted to the corrected data. As one can see, the data is within the errors compatible with the curve given by the Neĭmark-Sacker dynamics.

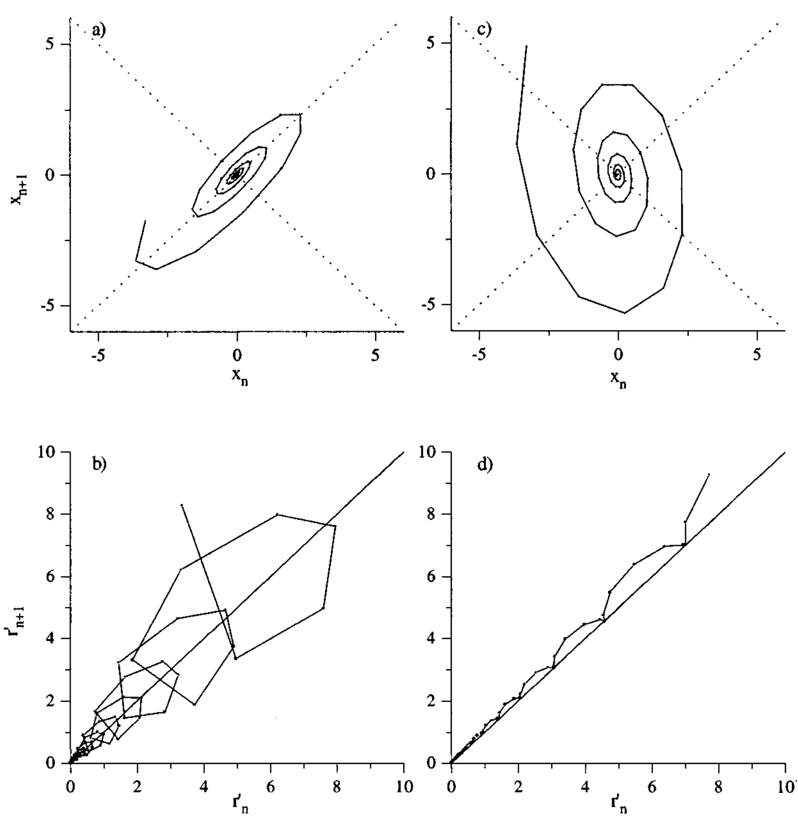

FIG. 12. a) First-return map reconstructed according to Eq. (6) and (b) corresponding radius dynamics; (c) and (d) show corrected versions of (a) and (b).

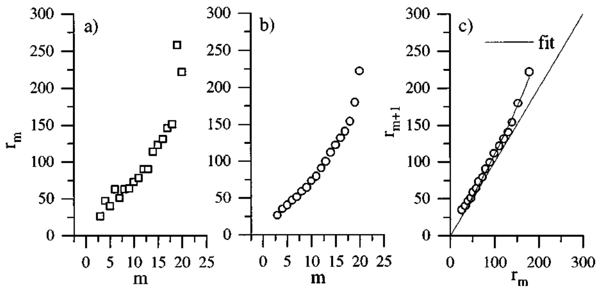

FIG. 13. (a) Original and (b) smoothed r_m and (c) radius dynamics for smoothed data of (b) including the Neĭmark-Sacker fit [Eq. (3)].

VI. DISCUSSION

In Sec. V it was shown that different analyses of the data obtained from the experimental system under investigation are in agreement with the interpretation of the presence of type II intermittence.

However, it has to be mentioned that these analyses are not rigorous proof for intermittence II, it was solely shown that the data do not contradict the interpretation of intermittence type II. This is a general problem of experimental data and not specific to the present system under investigation. But as there are four different aspects that have all been shown to be compatible with this kind of intermittence, there is some evidence that type II is really present in this case.

Since up to now no theoretical model to describe the system could be formulated, it is impossible to cross-check the experimental results by numerical calculations.

ACKNOWLEDGMENTS

We are grateful to Professor W. Kreische for helpful and stimulating discussions and his encouragement.

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